On the coherence veryfication of the continuous variable state in Fock space

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Abstract

In quantum optics, all novel observable phenomena are cased by the real quantum coherence. However as it is pointed out by Rudolph and Sanders[1] the phase information in a continuous variable(CV) state in the Fock space has never been shown so far. This may cause serious problem to many conclusions in quantum optics. In particular, a type of CV states, two mode squeezed states are used as the entanglement resource for the quantum teleportation(CVQT) experiment[2]. Verification the coherence or incoherence for an ensemble of two mode squeezed states from the conventional Laser is crucial to the validity of quantum teleportation experiment with CV states. This coherence is determined by the relative phase inside of a state. Unfortunately, so far all the quantum tomography results are insensitive to the phase information of the state. Here we give a simple scheme to distinguish two different ensemble of states: the Rudolph-Barry ensemble, which consists of squeezed states with uniform distribution of the phase; and the Enk-Fuchs ensemble, which consists of identical states with a fixed(but unknown) phase for every state. We believe our proposal can help to terminate the recent argument on the validity of CVQT with a clear conclusion.

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Quantum coherence of different states plays a fundamentally important role in the whole subject of quantum optics. The squeezed states have been widely used to demonstrate various novel non-classical properties in the past. In particular, a two mode squeezed state

$$|r,\phi>= e^{re^{i\phi}(a_1a_2-a_1^\dagger a_2^\dagger)}|00\rangle, \tag{1}$$

where $a_i^\dagger$ and a_i are bosonic creation and annihilation operators respectively in the two mode Fock space, $|00>$ is the vaccum state, is hopefully to be used as the entangled resource to carry out various novel tasks in the creteria of quantum information[6] such as the quantum teleportation[7, 8]. Recently, it has been used as the enatnglement resource to teleport a quantum state between two spatially separated parties[2]. However, since the phase ϕ in the state has never been tested, the states as defined by the above equation from certain source could have different phase ϕ for each different wavepackets. If this is the case, the lack of the phase information causes the loss of coherence of the state. Although lots of experiments have been done sucessfully to reconstruct the continuous variable states in the past, none of them is related to the phase information. That is to say, a pure state is not the only possible state compatible with the observed results. As it is noted[1] that, conventional meassurement on optical fields, such as homodyne detection using lasers, involving mixing of defferent incoherent fields and subsquent detection by energy absorption in photodetectors: all such meassurements are completely insensitive to any optical coherence. Actually, the state could be in arbitrary classical probabilistic distribution via the different phases. If we use the random uniform distribution, the observed state is actually a mixed diagonal state in Fock space[1, 9] which is given by

$$\frac{1}{2\pi}\int_0^{2\pi}|r,\phi><r'\phi|d\phi = \frac{1}{1-\tanh^2 r}\sum_0^\infty \tanh^{2n} r|nn><nn|. \tag{2}$$

Definitely, there is neither quantum coherence nor quantum entanglement in the above state. The state defined by eq(2) is a totally classical state. If the states from certain source indeed have random phases, the so called squeezed states will have little novel properties in the practical use because in cuch a case all quantum coherence has been lost and all the observable phenomena are just the same as that given by classical optics. For example, we can never really take the advantage of squeezing property for certain quadrature variable(although a certain quadrature operator ie indeed squeezed on a single wavepacket).

Due to the lack of the phase information, the observable quantum coherence property have never been verified on an ensemble of squeezed states. To answer the question that whether the nontrivil quantum coherent state which may cause novel observable quantum coherence phenomena really exists we must have a way to detect the phase information.

Recently, the phase information of the two mode squeezed states has drawn much attention of the physicists due to the issue of quantum teleportation of the continuous variable state in Fock space[2]. Since the tomography result is independent of the phase information ϕ, it is also possible that the the state used as the entanglement source in the quantum teleportation experiment is a mixture of the states with different ϕ, as defined by eq.(2).

If this is the case, then no quantum state can be teleported by such a separable state. Definitely, the an ensemble of N copies pure states as defined by $|r,\phi><r,\phi|^{\otimes N}$ is totally different from the case as defined by eq.(2). Unfortunetely, so far there has been no way to distinguish these two totally different cases.

Due to the unclearity of the phase ϕ, there is a very hot discussions on the validity of the entanglement resource used in the CVQT experiment[1, 3, 4, 5]. The discussions can be sumarrized as the following:

1. Rudolph-Sanders: An ensemble of squeezed states from a conventional laser are uniformly distributed on ϕ, the correct form of the quantum state for the ensemble is given by eq(2)

2. Enk-Fuchs: The traditional formalism is insufficient to describe the two mode squeezed, using quantum de Finetti theorem one can see that the two mode squeezed states produced by a conventional laser is essentially an ensemble of many coppies of identical two mode squeezed states with a fixd ϕ, though the value of ϕ is unknown.

In this letter we give a scheme to detect the phase information ϕ in the two mode squeezed state. That is to say, the meassurement result by our scheme is *sensitive* to the quantum coherence. Using our scheme, the states defined by eq(1) or eq.(2) can be easily distinguished. Obviously, the scheme has broad potential applications in the whole subject of quantum optics. For example, given many copies of pure squeezed states, we can verify that they are indeed pure states with the same phase ϕ. For another example, given two Fuch sources, each source emmits many identical two mode squeezed states, the phase of the state from each source ϕ_1 and ϕ_2 respectively, they are fixed and unknown, our scheme can be used detect the value $\phi_1-\phi_2$. As an immediate application, the scheme could be used to judge the state used as the entangled resource in a recent quantum teleportation[2] is the pure state as defined in eq.(1) or a mixed state defined by eq.(2). Consequently, this detection could give a a clear judgement on whether the experiment done in ref.[2] is essentially a quantum teleportation or a classical simulation of quantum teleportation.

Lets first see what happens if a photon source indeed emmits the identical pure squeezed states. As it is noted in ref[10] that a two mode squeezed state can be produced with the specifically chosen polarization for each modes as the following

$$|\psi_1(\phi)>=\sum_{l=0}^{\infty}\lambda^l e^{il\phi}|l>_{ah}|l>_{bv}, \tag{3}$$

where the subscripts a, b are for the mode a and b respectively, h and v represent for the horizontal and vertical polarizations respectively. Another wave packet from the same source must be in the same squeezed state, after the mode a and b is exchanged, $|\psi_2(\phi)>=\sum_{m=0}^{\infty}\lambda e^{im\phi}|m>_{av}|m>_{bh}$. The total state is then given by

$$|\Psi>=|\psi_1>|\psi_2>=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\lambda^{l+m}e^{i(l\phi+m\phi)}|l>_{ah}|m>_{av}|l>_{bv}|m>_{bh}\ . \tag{4}$$

After taking a meassurement on the photon number of each mode, the state $|\Psi>$ is collapsed to a specific entangled state. As it has been demonstrated in ref[10], tgis type of meassurement cab be carried out by either either a quantum non-demolition meassurement[11] or by a more feasible way, the destructive photon counting. Suppose the meassurement result is n for each mode, then the state after meassurement is $\sum_{m=0}^{n} e^{i[(n-m)\phi+m\phi]}|n-m>_{ah}|m>_{av}|n-m>_{bv}|m>_{bh}$. The phase information is clearly included in the state after the meassurement. For simplicity, we consider only the cases of $n=1$ only. In the real experiment we can choose an appropriate value of λ so that we have a significant probability to get the result of $n=1$, thus given many coppies of state $|\Psi>$, we can always have a significant number of states with $n=l+m=1$ after the meassurement. In the case of $n=1$, the state is

$$|\Psi_1>=e^{2i\phi}(|1>_{ah}|0>_{av}|1>_{bv}|0>_{bh})+|0>_{ah}|1>_{av}|0>_{bv}|1>_{bh}\ . \tag{5}$$

There is only one photon in each mode for the satte defined above. Obviously, the state defined above can be rewritten in the following way

$$|\Psi_1>=e^{2i\phi}(|H>_a|V>_b+|V>_a|H>_b). \tag{6}$$

Where the states $|H>$, $|V>$ are for a horizontal or a vertical polarized photon states respectively. For this state, meassurement results for the polarization of the two modes are always the $different$. Clearly, if the states given from the source are indeed pure states, i.e., they are many coppies of $|\Psi_1>$, then we can rotate the polarizers to both modes, and in principle we can find certain angle($\pi/4$) by which the meassurement result of the polarization in the two modes are always the *same* ! On the other hand, if the given states from the source are mixed states as defined in eq(2), i.e., phase ϕ in each wave packet can be different. In such a case, by the same operation the state is $\frac{1}{2}(|HV><HV|+|VH><VH|)$, the correlation between the two modes will linearly decreased with $\cos^2\beta$ when the polarizers are rotated, here β is the angle that is rotated. Consequently, an Enk-Fuchs source and a Rudolph-Barry source can be distinguished in the following way:

Initially, in both cases they are totally negatively correlated, i.e., whenever a photon is detected in mode a, there most be no photon detected in mode b, and vice versa. We rotate the polarizers by $\pi/4$, Enk-Fuchs source(see eq(1)) will give a totally positive correlation, while the Rudolph-Barry source(see eq(2)) will give no correlation. Also, in rotating the polarizers, if the correlation does not decrease linearly with $\cos^2\beta$, the source must be not Rudolph-Barry one.

Obviously, the above scheme can be also used to detect the phase difference for two Enk-Fuchs sources. Now $|\psi_1>$ and $|\psi_2>$ are collected from two different sources whose unknown fixed phase are denoted by ϕ_1 and ϕ_2 respectively. After the measurement on the photon number basis two each mode is done, the state up to a global phase factor is

$$|\Psi_1>=|H>|V>+e^{i(\phi_2-\phi_1)}|V>|H>\ . \tag{7}$$

We rotate the polarizers continuously and test the correlations at each stage. There must be certain angle β_0 at which we can observe that the meassurement results of the two modes are totally positively correlated. This angle β_0 determines the phase difference $\phi_2-\phi_1$.

Note that in a real experiment, we actually do not need to take a meassurement on the basis of of the photon numbers of each mode, i.e. $\sum_{n=0}^{\infty} |n >< n|$, where $n = l + m$. The meassure on such a basis could be difficult by our current technology. The only dificulty is to collect the wave packet $|\psi_1 >$ and $|\psi_2 >$ so that they are indistiuguishable. Once we can make $|\psi_1 > |\psi_2 >$ indistinguishable we can simply carry out the detection scheme by the usual photon counting and obtain the correct result to a good approximation. Suppose we use port A and port B to detect the photons from mode A and B respectively. A polarizer is placed before each port. The polarizers are placed horizontally in the begining. We choose a very small r so that $r^2 << 1$ for the photon source. In the photon counting, we only record the data of those events where at least one port detects one photon *and* no port detects more than one photon. For simplicity, we name the events satisfying this condition as "good events". We will analyse the correlation between the two modes only using the data of the good events. When the polarizers are rotated by certain angle β, a good event could be a event caused by a state with $l + m \neq 1$, i.e., the wave packet could have collapsed to a state of which the photon number of each mode is not 1. It's easy to see that this type of event can only happen with a small ptobability. First, $l+m = 0$ is impossible, because whenever we observed a good event, we have observed one photon at least in one port, so the photon number of that mode must not be 0. Second, $l + m = 2$ or a larger number is possible but the chance is negligible. We have already set r to be very small. Since $r^2 << 1$, the probability that the state $|\Psi >$ collapsed to a state with $l + m = 1$ is much larger than that of a state with $l + m > 1$. For example, taking $r = 0.01$, in average, we can have one good event from 10000 events, and the probability that this good event is caused by a stete with $l + m = 1$ is several thousands times larger than the that by a state with $l + m \neq 1$. Initially, all good evnts must be the cases that 1 photon detected in certain mode and 0 photon is detected in another mode. If all wavepackets are from one Enk-Fuch source, whent the polarizers are rotated by $\pi/4$, 1 photon is detected in each mode to all good events. If they are from the Rudolph-Barry source, when the polarizers are rotated by $\pi/4$, too all good events, half of them are $(1_A, 0_B)$ or $(0_A, 1_B)$ and half of them are $(1_A, 1_B)$, where the 0 or 1 represents the number of photons detected, subscripts A, B represent the mode A, B respectively. The phase difference can be detected for two Enk-Fuchs sources. In this case, we have to rotate the polarizers continuously. The phase difference $\phi_2 - \phi_1$ is determined by the angle β_0 we observed 1 photon in each mode to all good events. Thus we see, to a good approximation, our scheme can carried out by the normal photon counting technique.

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